13.4.5 Interactions of Migrating Point Defects With Dislocations

Two things to consider here;

1.) Capture of intrinsic defects (vacancies, interstitials). This causes dislocations to move.

2.) Capture of gas atoms (this may cause dislocations to be pinned).

Capture radius ---- Figure 13.6

Since the number of capture sites per cm$^3$

\[ Z_{gt} C_t \]

Sites per trap Trapping centers

\[ \frac{Z_{vd}}{a_o \rho_d} \]

Per unit length of dislocation

\[ 1 \]

\[ \frac{Z_{vd} \rho_d}{\rho_d} \]

Rate of vacancy capture by dislocations/ cm$^3$ = \( D_V Z_{vd} \rho_d C_V \)

Rate of interstitial capture by dislocations/ cm$^3$ = \( D_I Z_{id} \rho_d C_i \)

Capture rate of gas atoms by dislocations/ cm$^3$ = \( D_{Xe} Z_{gd} \rho_d C \)
13.5 Diffusion Limited Reactions

Interaction rates have two components
1.) Diffusion in the bulk to the defect
2.) Reaction rate at the surface of the defect

13.5.1 Diffusion to Spherical Sinks

Set up unit cell - Figure 13.7

\[
\frac{\partial C}{\partial t} = \left\{ \frac{D}{r^2} \right\} \frac{\partial}{\partial r} \left( r^2 \frac{\partial C}{\partial r} \right) + Y \cdot F
\]

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<th>Time Variation</th>
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<tbody>
<tr>
<td>Variation</td>
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Note: Neglecting Recombination at equilibrium,

\[
\frac{d}{dr} \left( \frac{r^2 dC}{dr} \right) = -Y F
\]

Noting:
\[
\left( \frac{\partial C}{\partial r} \right)_R = 0
\]

and
\[
C(R, t) = C_R
\]

Then;

\[
C(r) = C_R + \frac{Y F}{6D} \left[ \frac{2R^2(r-R)}{rR} - \left(r^2 - R^2\right) \right]
\]

Assume \( R >> R \), Figure 13.8

Region 1 (neglect production rate)

\[
\frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2 dC}{dr} \right) = 0
\]

and \( C(\infty) = C(R) \)
Solving;

\[ C(r) = C_R + [C(\mathbb{R}) - C_R] \cdot \left[ 1 - \left( \frac{R}{r} \right) \right] \]

Since

\[ J = -D \left( \frac{dC}{dr} \right)_R \]

\[ J = -D \left[ \frac{C(\mathbb{R}) - C_R}{R} \right] \]

**Two parameters of Importance**

1.) Absorption by a sphere

\[ = - (4\pi R^2) J \]

\[ = 4\pi DR[C(\mathbb{R}) - C_R] \]

**What is this?**

\[ \frac{4\pi}{3} (\mathbb{R}^3 - R^3) Y F = 4\pi RD[C(\mathbb{R}) - C_R] \]

\[ C(\mathbb{R}) = C_R + \frac{Y F \mathbb{R}^3}{3DR} \]
2.) Absorption by all spheres

\[ \text{if } C(\mathcal{R}) \gg C_R \]
\[ = 4pRDCC_t \]

rate constant
\[ k = 4pRD \]

---

see page 213 for derivation of bubble radius during post Irradiation annealing

\[
\ln \left( \frac{R_f + R}{R_f - R} \right) = \frac{3D_{Xe}R_ft}{\mathcal{R}^3}
\]

Note : only good for ideal gas

13.5.2 Diffusion to Dislocations

1.) Switch to cylindrical co - ordinates
\[ (p\mathcal{R}^2)r_d = 1 \]

2.) At equilibrium;

\[
\frac{D_v}{r} \left[ \frac{d}{dr} \left( r dC_v \right) \right] = -Y_{vi} \cdot F + \kappa_{vi} C_v C_i
\]

recombination

approximate this by

\[
\left( Y \cdot F \right)_{eff} = Y_{vi} \cdot F - \kappa_{vi} C_v C_i
\]
3.) Exact Solution

\[ C_v(r) = C_{Rd} + \left( \frac{Y F}{2D_v} \right)_{\text{eff}} \frac{\mathcal{R}^2}{2} \ln \left( \frac{r}{R_d} \right) - \frac{1}{2} \left( \frac{r^2 - R_d^2}{\mathcal{R}^2} \right) \]

4.) Approximate Solution

\[ C_v(\mathcal{R}) = C_{Rd} + \left( \frac{Y F}{2D_v} \right)_{\text{eff}} \frac{\mathcal{R}^2}{2} \ln \left( \frac{\mathcal{R}}{R_d} \right) - \frac{1}{2} \]

5.) Rate of Vac. Capture by Dislocations/ cm\(^3\)

\[ = \left[ \frac{2\pi D_v \rho_d C_v}{\ln \left( \frac{\mathcal{R}}{R_d} \right)} \right] \quad \text{This is all diffusion controlled} \]

13.5.3 Mixed Rate Control

For a reaction rate control to dislocations, the vacancy capture/ cm\(^3\)

\[ = D_v Z_{vd} r_d C_v \]

Analogous to heat conduction, use vacancy capture/cm\(^3\) to be, (in the intermediate regime)
\[
\text{Reaction Rate} \quad \text{Diffusion (for } r_d \approx 10^{10}) \\
\approx 0.04 \quad \approx 0.7
\]

Hence, the vacancy absorption is almost entirely diffusion controlled
Diffusion Limited

Reaction Rate Limited

Fission Fragment
Problem 13.6

• In a fuel pellet we have N bubbles/ cm\(^3\) of radius R plus one bubble of radius R*.

• Both bubble sizes are in equilibrium and large enough to use perfect gas laws.

• When R* exceeds R\(_c^*\), the large bubble can gobble up small bubbles and grow spontaneously.

• Determine R\(_c^*\) (critical radius)

• If R\(_c^*\) = 10R, what is swelling at breakaway?

Condition of mechanical equilibrium means

\[ p = \frac{2\gamma}{r} \quad \text{and} \quad p \left( \frac{4\pi r^3}{3} \right) = mkT \]

or,

\[ m = \left[ \frac{8\pi\gamma}{3kT} \right] r^2 = C r^2 \hspace{1cm} 1) \]

If the large bubble expands by dR*, it sweeps out a volume \(4\pi R^{*2} dR^*\).

This volume contains N bubbles of radius R per cc and each bubble contains

\[ m = CR^2 \quad \text{atoms} \]
Therefore, the number of additional gas atoms acquired by the large bubble as a result of expansion by \( dR^* \) is:

\[
\text{dm}^* = 4\pi R^{*2} \cdot dR^* \cdot N \cdot C \cdot R^2 \quad 2)
\]

However, to maintain mechanical equilibrium according to eq. 1), the number of additional gas atoms required is given by:

\[
\text{dm}^* = \frac{d(CR^{*2})}{dR^*} \cdot dR^*
\]

\[
= 2CR^* \cdot dR^* \quad 3)
\]

The bubble will grow spontaneously if

\[
\text{dm}^* > \text{dm}^*
\]

or if

\[
4\pi R^{*2} \cdot N \cdot C \cdot R^2 > 2CR^*
\]

or,

\[
R^* \geq \frac{1}{2\pi NR^2} \quad 4)
\]

Now if \( \frac{R^*}{R} = 10 \), then from eq. 4

\[
N = \frac{1}{2\pi \cdot 10 \cdot R^3}
\]
But

\[ V_{\text{bubble}} = \frac{4\pi R^3}{3} \]

\[ \therefore N V_{\text{bub}} = \Delta V = \frac{3}{2\pi \cdot 10 \cdot R^3} = \frac{2}{30} = 0.0667 \]

Swelling \[ \frac{\Delta V}{V} = \frac{0.0667}{1 - 0.0667} = \frac{0.0667}{0.9333} = 7.2\% \]