### Chapter 14
Pore Migration and Fuel Restructuring Kinetics

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**Major Differences Between Pores and Bubbles**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pores</th>
<th>Bubbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>Large, &gt;1 mm</td>
<td>Small ( \approx 0.01 \text{ mm} )</td>
</tr>
<tr>
<td>Gas</td>
<td>He, CO, CO\textsubscript{2}</td>
<td>Fission Gases</td>
</tr>
<tr>
<td>Gas Pressure</td>
<td>Low, Few Atm.</td>
<td>High, 100’s ATM.</td>
</tr>
<tr>
<td>Densification</td>
<td>Important</td>
<td>Not Applicable</td>
</tr>
<tr>
<td>Shape</td>
<td>Lenticular</td>
<td>Spherical</td>
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<tr>
<td>Movement</td>
<td>Vapor Transport</td>
<td>Surface or Volume Diffusion</td>
</tr>
<tr>
<td>Diffusion</td>
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</table>

**Bubble Migration**

- Surface Diffusion \( \propto \frac{1}{R} \)
- Volume Diffusion \( \propto \text{constant} \)

**Pore Migration**

- Vapor Transport \( a \) \( R \) (mech. equilibrium.)
- See Figures 14.1, 14.2
14.2 Pore Migration by Vapor Transport

Figure 14.5

Set heat transfer parameters equal

\[ k_p \left( \frac{dT}{dx} \right)_p = k_s \left( \frac{dT}{dx} \right)_p \]

and \( \frac{k_s}{k_p} \approx 5 \)

Note; \( v_p \propto J \Omega \)

\[ J = \left\{ \frac{D_g}{kT\delta} \right\} \left[ P^\circ(x + \delta) - P(x) \right] \left( \frac{dP^\circ}{dT} \right) \left( \frac{dT}{dx} \right)_p \delta \]

Diff. Coeff. for matrix molecules in gas

\[ D_g = \left( \frac{\text{Const.}T^\frac{3}{2}}{\sigma_{12}^2\Omega_{DP}} \right) \sqrt{\left( \frac{M_1 + M_2}{M_1M_2} \right)} \]

Collision diam.

for 2 species in gas

Parameter from theory

\[ \propto \frac{kT}{\varepsilon_{12}} \]

Force constant
Two things to consider;

1.) Impurities (Fig. 14.6)
   *They depress vapor pressure on the hot side.*

2.) Cold Side Condensation Limitations
   *Increases vapor pressure*

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14.3 Porosity Redistribution Kinetics
   Read section for model description
   (Figures 14.8, 14.9)

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14.4 Columnar Grain Growth

*Using the approach by Nicols, get pore velocity as a function of fuel radius. See figure 14.10.*

*Define d= distance a pore at outer edge of columnar zone moves into zone.*

\[
t = - \int_{r_1}^{r_1-d} \frac{dr}{v_p(r)} = - \int_{T_1}^{T_1} \frac{dT}{v_p\left(\frac{dT}{dr}\right)}
\]

outside of columnar grain region

*Rest of section involved with above equations for t with appropriate values of v_p*

Problem: Define the fractional radius of central void in terms of initial porosity
14.5 Equiaxed Grain Growth Region

- Curved grain boundaries cause large grains to grow at the expense of small grains

Figure 14.12

Atoms like to be on the concave side instead of convex side because they are surrounded by more matrix atoms

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- Kinetics

\[ d^m - d^m_0 = k_0 t \exp \left( \frac{Q}{kT} \right) \]

\[ d^2 - d^2_0 = k_0 t^a \exp \left( \frac{Q'}{kT} \right) \]
Problem 14.1

During operation of a fresh fuel pin, all pores within 0.8 R migrate toward the center void.

a.) Calculate radius of central void if initial porosity is $P_o$

b.) Determine $T(r)$ in $r_o < r < 0.8 R$

• assume $k$ independent of $T$
• assume flat power density appropriate to $r_o$.

c.) Assume $v_p(T)$ is known, how would you calculate time to form a central void

d.) In $v_p(T)$ of c.), use a gas pressure $p$ which reflects the collection of gas in the path of the pore.

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a.) Calculate $r_{of}$

$$\pi (0.8R)^2 P_o = \pi r_{of}^2$$

$$\frac{r_{of}}{R} = 0.8 \sqrt{P_o} \hspace{1cm} 1.$$ 

b.) For constant $k$, the heat conduction in the fuel is:

$$\frac{k}{r} \left[ \frac{d}{dr} \left( \frac{r dT}{dr} \right) \right] = -H \hspace{1cm} 2.$$
use $T(R) = T_s$ and $\left( \frac{dT}{dr} \right)_{r_o} = 0 \quad 3.), 4.)$

If the rod is operated at constant linear power, the average heat generation rate in the columnar grain region is (if the restructuring takes place in that zone only);

$$\bar{H} = \frac{H_o}{1 - \left( \frac{r_o}{0.8R} \right)^2} \quad 5.)$$

where $H_o$ is the initial power density of the fuel

Since $r_o$ starts at 0 and increases to value given above, $\frac{r_o}{R}$ increases with time and therefore $\bar{H}$ increases with time.

Assume that local power density is equal to average power density throughout the region (H in equation 2 is equal to $\bar{H}$ given by eq 5 for all $r_o < r < 0.8R$)

$$T_1(r) = -\frac{[\bar{H}r^2]}{4k} + A \ln r + B \quad 6.)$$

From boundary condition 4

$$A = \left[ \frac{\bar{H}r_o^2}{2k} \right] = \left[ \frac{H_o R^2}{2k} \right] \left( \frac{\bar{H}}{H_o} \right) \left( \frac{r_o}{R} \right)^2 \quad 7.)$$
Heat Generation Rate in $0.8R < r < R$

In the unrestructured region; $r > 0.8R$

$$T_2(r) = -\left(\frac{H_0r^2}{4k}\right) + C \ln r + D \quad \text{8.)}$$

Using B. C. (3)

$$T_s = -\left(\frac{H_0R^2}{4k}\right) + C \ln R + D \quad \text{9.)}$$

If we match the results at $r=0.8 \, R$

$$T_1(0.8R) = T_2(0.8R) \quad \text{11.)}$$
and

$$\left(\frac{dT_1}{dr}\right)_{0.8R} = \left(\frac{dT_2}{dr}\right)_{0.8R} \quad \text{12.)}$$

Note it is assumed that $k$ is constant throughout

$$r_o < r < R$$

Using (6) and (8) in (11) yields;

$$-\left(\frac{H}{4k}\right)(0.8R)^2 + A \ln(0.8R) + B =$$

$$= -\left(\frac{H_0}{4k}\right)(0.8R)^2 + C \ln(0.8R) + D \quad \text{13.)}$$
using (6) and (8) in (12) yields

\[-\left(\frac{\bar{H}}{2k}\right)(0.8R) + \left(\frac{A}{0.8R}\right) = -\left(\frac{H_o}{2k}\right)(0.8R) + \left(\frac{C}{0.8R}\right)\]

Solving (14) for \(C\) gives

\[C = A - \left\{\frac{(\bar{H} - H_o)}{2k}\right\}(0.8R)^2 = A - \left(\frac{H_oR^2}{2k}\right)(0.64)\left(\frac{\bar{H}}{H_o} - 1\right)\]

Using eq 7 for \(A\) we get

\[C = \left(\frac{H_oR^2}{2k}\right)\left[\left(\frac{r_o}{R}\right)^2\left(\frac{\bar{H}}{H_o}\right) - 0.64\left(\frac{\bar{H}}{H_o} - 1\right)\right] \quad 16\]

Solving eq (9) for \(D\)

\[D = T_s + \left(\frac{H_oR^2}{4k}\right) - C \ln R \quad 17\]

where \(C\) is given by eq (16)

Using eq (13) and (17) to determine \(B\)

\[B = \left(\frac{\bar{H} - H_o}{4k}\right) \cdot (0.8R)^2 + (C - A)\ln(0.8R) + T_s + \left(\frac{H_oR^2}{4k}\right) - C \ln R\]

Substituting \(B\) into (6)
\[ T_1(r) - T_s = -\left( \frac{\bar{H}r^2}{4k} \right) + A \ln r + \left( \frac{H - H_o}{4k} \right)(0.8R)^2 \]

\[ + (C - A) \ln(0.8R) + \left( \frac{H_o R^2}{4k} \right) - C \ln R \]

Use

\[ C \ln R = [A + (C - A)] \ln R = A \ln R + (C - A) \ln R \]

and

\[ T_1 - T_s = -\left( \frac{\bar{H}r^2}{4k} \right) + \left( \frac{H_o R^2}{4k} \right) + \left( \frac{H - H_o}{4k} \right)(0.8R)^2 \]

\[ + A \ln \left( \frac{r}{R} \right) + (C - A) \ln(0.8) \]

for

\[ T_1 - T_s = -\left( \frac{H_o R^2}{4k} \right) \left\{ 0.36 + \left( \frac{\bar{H}}{H_o} \right) \left( 0.64 - \left( \frac{r^2}{R^2} \right) \right) \right\} \]

\[ + \left( \frac{H_o R^2}{2k} \right) \left( \frac{H}{H_o} \right) \left( \frac{r_o}{R} \right)^2 \ln \left( \frac{r}{R} \right) \]

\[ - \left( \frac{H_o R^2}{2k} \right) \left[ 0.64 \ln(0.8) \right] \left( \frac{H}{H_o} - 1 \right) \]

Eq 5 \[ \frac{\bar{H}}{H_o} = \frac{1}{1 - \left( \frac{r_o}{0.8R} \right)^2} \]

and \[ H_o = \frac{\Re}{\pi R^2} \]
\[ T_1 - T_s = \frac{\kappa}{4\pi k} \left\{ 0.36 + \frac{0.64 - \left( \frac{r}{R} \right)^2}{1 - \left( \frac{r_o}{R} \right)^2} \right\} \]
\[ + \frac{\left( \frac{r_o}{R} \right)^2}{1 - \left( \frac{r_o}{R} \right)^2} \cdot \left[ \ln \left( \frac{r}{R} \right) - 0.64 \ln(0.8) \right] \]  

Eq (18) gives the temperature distribution in the columnar grain region as a function of \( \left( \frac{r_o}{R} \right) \).

The central void radius starts at \( r=0 \) at \( t=0 \) and grows to \( r_{of} \) (eq 1) at the end of restructuring.

We assume we know \( \left( \frac{r_o}{R} \right) \) as a function of time.

Therefore the temperature distribution in the columnar grain region is a function of both \( r \) and \( t \), or \( T(r,t) \).
c.) Time to complete restructuring = Time for a pore at the outer edge of the columnar grain region to migrate to the final central void position
\[ r_o = r_{of} \]

The pore velocity is a known function of temperature, but as a result of the solution in b.), the temperature is a function of \( r \) and \( t \). Therefore, the velocity of a pore is a function of \( r \) and \( t \), or,

\[ v_p(r,t) \]

Let \( r \) = radial position of a pore which started at \( r=0.8R \) at \( t=0 \).

\[ v_p(r,t) = \frac{dr}{dt} \quad 19) \]
\[ r(0) = 0.8R \quad 20) \]

Eq. 19 must be integrated numerically, obtaining \( v_p \) as a function of \( T \) from pore migration theory and \( T_1(r,t) \) from (18)

Eq (19) must be integrated from \( r=0.8R \) up to \( r = r_{of} = 0.8R \sqrt{P} \). The time at which this is reached is \( t_f \).

d.) The pore velocity is given by a combination of eqs (14.6) and (14.9)
\[ v_p(T, p) = \left( \frac{C}{pT^2} \right) \exp\left( -\frac{\Delta H_{vap}}{RT} \right) \]  

where \( C \) is a constant

In the as fabricated (cold) fuel, the number of gas atoms in each pore is given by;

\[ p_0 \left( \frac{4\pi r^3}{3} \right) = mkT_a \]  

When heated to a temperature \( T \) and subject to the condition of mechanical equilibrium;

\[ p \left( \frac{4\pi r^3}{3} \right) = mkT \]  

\[ p = \frac{2\gamma}{r_p} \]  

dividing (22) by (23) we get

\[ \left[ \frac{p_0}{p} \right] \cdot \left( \frac{r_{po}}{r_p} \right)^3 = \frac{T_a}{T} \]  

Using \( r_p \) from 24 and 25;
\[ p = \sqrt{\frac{T_a}{p_o T \cdot \left( \frac{2\gamma}{r_{po}} \right)^{\frac{3}{2}}}} \quad (26) \]

substituting (26) into (21) gives

\[ v_p (T) = C \sqrt{\frac{p_o}{T_a}} \cdot \left( \frac{r_{po}}{2\gamma} \right)^{\frac{3}{2}} \cdot \exp \left( - \frac{\Delta H_{vap}}{RT} \right) \cdot \frac{1}{T} \]