Enrichment and Conversion of Fission Reactor Fuel Elements

- Two fissile isotopes commonly considered:
  
  - $^{235}U$ (Use enrichment)
  - $^{239}Pu$ (Use reprocessing)

- U.S. (weapons---->submarines---->civilian 1944 present)

- Canada..Commercial..Heavy Water + Nat. U

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Gaseous Diffusion Process - $UF_6$

- Relies on the fact that $UF_6$ is solid at RT and a vapor at moderate temperatures.

(Figure)

- Gaseous diffusion relies on the difference in the rate which $^{235}UF_6$ and $^{238}UF_6$ diffuse through a barrier containing many holes.

- The relative speed of the two molecules can be derived from their kinetic energies:

$$kT = \frac{MV^2}{2}$$

M = molecular mass

or,

$$\frac{V_L}{V_H} = \sqrt{\frac{M_H}{M_L}} = \alpha$$
• Relative frequency at which molecules of different species pass through a small hole is proportional to the speed of the molecule.

Hence the ratio of \( \frac{^{35}U}{^{38}U} \) on the low pressure side is greater than the \( \frac{^{35}U}{^{38}U} \) ratio on the high pressure side.

For \(^{235}UF_6\) and \(^{238}UF_6\), maximum \( \alpha \) is:

\[
\alpha = \sqrt{\frac{238 + 6 \sum 19}{235 + 6 \sum 19}}
\]

\( \alpha = 1.004289 \)

( more realistic value is 1.003 due to down stream back pressure and leaks)

• Low value of \( \alpha \) requires a very large number of steps

Figures (2)

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• What is the Barrier? (mostly classified)
  • Very thin and delicate
  • 100's of millions of holes/cm³
  • \( \approx 20 \) Å diameter hole
  • Must exclude organic materials and air to avoid plugging
  • Materials reported to be sintered Ni and anodized Al
More complete analysis of U enrichment, see:


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**Important Variables and Equations**

\[ \text{kg U Feed (F)} = \text{kg Enriched U Product (P)} + \text{kg U Waste (W)} \]

\[ x^F_f F = x^P_p P + x^W_w W \]

where

- \( x_f \) = wt. fraction of \( ^{35}U \) in feed
- \( x_p \) = wt. fraction of \( ^{35}U \) in product
- \( x_w \) = wt. fraction of \( ^{35}U \) in waste

(Note: F, P, & W could be in kg or kg/unit time)

- 2 eqs. and 6 variables, F, P, W, \( x_f \), \( x_p \), \( x_w \)
- Trick is to solve for 2 in terms of the other 4!

1.) \( x_f = 0.711\% \) now (1996)

2.) \( x_p \) = as requested by the customer

Table 1.1 and 2 figures

3.) \( x_w \) = could be between 0.2 and 0.3 \%, currently in the U.S. is 0.3\%
4.) \( P = \text{mass of desired product} \)

One can solve the equations above;

\[
F = P \left( \frac{x_p - x_w}{x_f - x_w} \right)
\]

\[
W = P \left( \frac{x_p - x_f}{x_f - x_w} \right)
\]

**Feed factor is defined as;**

\[
\frac{F}{P} = \left( \frac{x_p - x_w}{x_f - x_w} \right)
\]

**Waste factor ;**

\[
\frac{W}{P} = \frac{F}{P} - 1
\]

How Much Energy is Required to Reach a Given Enrichment?

Define Separative Work Unit (SWU) as;

"resource required to perform the enrichment to the desired level of \( x_p \) given \( x_f \) and \( x_w \). For gaseous diffusion this is equivalent to electrical energy"

# of SWU's produced by an enrichment plant
during a time period \( t \),

\[
SWU = \left[ P \sum V(x_p) + W \sum V(x_w) - F \sum V(x_f) \right] t
\]

The quantity \( V(x_i) \) is called the separation potential and is given by;

\[
V(x_i) = (2x_i - 1) \ln \left( \frac{x_i}{1 - x_i} \right)
\]

where \( i = f,p,w \)

We normally quote SWU's per unit of product (\( P \cdot t \)) where \( P \) is feed rate.

\[
S = \frac{SWU}{P \sum t} = V(x_p) + \left( \frac{W}{P} \right) \sum V(x_w) - \left( \frac{F}{P} \right) \sum V(x_f)
\]

\( S \) = "SWU" factor, \( \frac{SWU}{kg} \)

Figure 3.6 plus Schematic

Problem -1
a.) What is the number of kgs of natural U that has to be provided as feed in an enrichment plant if one requests 30,000 kg of U enriched to 3% in $^{35}U$? Assume the tails assay is 0.2%.

b.) What is the number of SWU’s needed for separation?

$$\frac{F}{P} = \frac{(3 - 0.2)}{(0.711 - 0.2)} = 5.479 \frac{kg \text{ feed}}{kg \text{ product}}$$

Total feed is then;

$$F = 30,000 \cdot 5.479 = 164,370 \text{ kg U feed}$$

b.)

$$V(x_f) = (2 \sum 0.00711 - 1) \ln \left[ \frac{0.00711}{1 - 0.00711} \right] = 4.869$$

$$V(x_w) = (2 \sum 0.002 - 1) \ln \left[ \frac{0.002}{1 - 0.002} \right] = 6.188$$

$$V(x_p) = (2 \sum 0.03 - 1) \ln \left[ \frac{0.03}{1 - 0.03} \right] = 3.268$$

$$S = 3.268 + (5.479 - 1)(6.188) - (5.479)(4.869) = 4.307$$

Hence the total number of SWU’s is then;

$$30,000 \text{ kg} \cdot 4.307 \text{ SWU/kg} = 129,210 \text{ SWUs}$$
<table>
<thead>
<tr>
<th></th>
<th>BWR</th>
<th>PWR</th>
<th>HTGR</th>
<th>CANDU</th>
<th>LMFBR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MW(e)</strong></td>
<td>1100</td>
<td>1100</td>
<td>330</td>
<td>508</td>
<td>1200</td>
</tr>
<tr>
<td><strong>Thermal Eff.-%</strong></td>
<td>33</td>
<td>33</td>
<td>39</td>
<td>30</td>
<td>40</td>
</tr>
</tbody>
</table>
| **Assembly Geometry**    | 8x8 | 9x9 | 17x17 | Hexagonal | Cylindrical | Hexogona
| **Assembly Length-m**    | 3.8 | 3.7 | 0.78  | 0.5   | 1     |
| **# of Assemblies**      | 590 | 180 | 1482  | 4680  | 360   |
| **Core Ht-m**            | 3.8 | 3.7 | 4.75  | 5.95  | 1     |
| **kg Fuel /assembly**    | 270 | 600 | 22    | 37    | 80    |
| **Tot.tonne fuel in core** | 138 | 90-100 | 0.77-235U | 105  | 29    |
| **BU-MWd per MTU**       | 30,000 | 30,000 | 100,000 | 8,000 | 100,000 |
| **% Fuel Replaced/y**   | 25  | 33  | 18    | continuous | Varied   |
| **Enrichment-%**        | 1.8 | 2.8 | 93    | 0.711 | 15-20 |
| **Power Density-(kW/liter)** | 54 | 100 | 8    | 12    | 280  |
| **Linear Ht Rate-kW/m** | 19 | 17  | 8    | 26    | 29    |
| **a-Initial Loading**    |     |     |      |       |       |
1.) An enrichment plant has a throughput of 32,000 kgU/day and produces 26,000 kgU as tails. What is the enrichment of the product if the feed is natural U and the tails are 0.25%?

2.) A gaseous diffusion method has been proposed to produce BF$_3$ enriched to 90% in B$^{10}$. How many kgs of BF$_3$ feed (natural B) are needed to produce 1 kg of B$^{10}$ with 8% tails?

3.) Calculate the natural U feed and SWU factors 1 billion years into the future. Assume tails of 0.15% and 3% enriched product;

$$t_{1/2} (^{35}\text{U}) = 7.1 \times 10^8 \text{ y},$$
$$t_{1/2} (^{38}\text{U}) = 4.51 \times 10^9 \text{ y}.$$

4.) Assuming that the price per SWU is $80 and the cost of conversion is $4/kgU, what is the price of the U$_3$O$_8$ ($\text{lb U}_3\text{O}_8$) beyond which it will cost less to enrich the already mined, purified, and converted (to UF$_6$) tails that contain 0.2% $^{35}\text{U}$ rather than mine new U?

[Assume the product will be 3% enriched in either case and the new tails will be 0.1% (when the old tails are enriched). Tails stored as UF$_6$ cost nothing.]