Enrichment and Conversion of Fission Reactor Fuel Elements

- Two fissile isotopes commonly considered:
  - $^{235}\text{U}$ (Use enrichment)
  - $^{239}\text{Pu}$ (Use reprocessing)

- U.S. (weapons---->submarines---->civilian )
  1944 present

- Canada..Commercial..Heavy Water + Nat. U

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Gaseous Diffusion Process - $\text{UF}_6$

- Relies on the fact that $\text{UF}_6$ is solid at RT and a vapor at moderate temperatures.

(Figure)

- Gaseous diffusion relies on the difference in the rate which $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$ diffuse through a barrier containing many holes.

- The relative speed of the two molecules can be derived from their kinetic energies;

$$kT = \frac{MV^2}{2} \quad M = \text{molecular mass}$$

or,

$$\frac{V_L}{V_H} = \sqrt{\frac{M_H}{M_L}} = \alpha$$
• Relative frequency at which molecules of different species pass through a small hole is proportional to the speed of the molecule. Hence the ratio of $\frac{^{35}U}{^{38}U}$ on the low pressure side is greater than the $\frac{^{35}U}{^{38}U}$ ratio on the high pressure side.

For $^{235}UF_6$ and $^{238}UF_6$, maximum $\alpha$ is:

$$\alpha = \sqrt{\frac{(238 + 6 \sum 19)}{235 + 6 \sum 19}}$$

$$\alpha = 1.004289$$

( more realistic value is 1.003 due to down stream back pressure and leaks)

• Low value of $\alpha$ requires a very large number of steps

Figures (2)

• **What is the Barrier?** (mostly classified)
  • Very thin and delicate
  • 100's of millions of holes/cm3
  • $\approx 20 \text{ Å}$ diameter hole
  • Must exclude organic materials and air to avoid plugging
  • Materials reported to be sintered Ni
  and anodized Al
More complete analysis of U enrichment, see;


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**Important Variables and Equations**

\[ \text{kg U Feed (F)} = \text{kg Enriched U Product (P)} + \text{kg U Waste (W)} \]

\[ x_f^F F = x_p^P P + x_w^W W \]

where \( x_f \) = wt. fraction of \(^{35}\)U in feed

\( x_p \) = wt. fraction of \(^{35}\)U in product

\( x_w \) = wt. fraction of \(^{35}\)U in waste

(Note: F, P, & W could be in kg or kg/unit time)

- 2 eqs. and 6 variables, F, P, W, \( x_f \), \( x_p \), \( x_w \)
- Trick is to solve for 2 in terms of the other 4!

1.) \( x_f = 0.711\% \) now (1996)

2.) \( x_p \) = as requested by the customer

   Table 1.1 and 2 figures

3.) \( x_w \) = could be between 0.2 and 0.3 %, currently in the U.S. is 0.3%
4.) \( P = \) mass of desired product

One can solve the equations above;

\[
F = P \left( \frac{x_P - x_w}{x_f - x_w} \right)
\]

\[
W = P \left( \frac{x_P - x_f}{x_f - x_w} \right)
\]

**Feed factor is defined as;**

\[
F = \frac{P}{P} \left( \frac{x_P - x_w}{x_f - x_w} \right)
\]

**Waste factor ;**

\[
W = \frac{F}{P} - 1
\]

How Much Energy is Required to Reach a Given Enrichment?

Define Separative Work Unit (SWU) as;

"resource required to perform the enrichment to the desired level of \( x_P \) given \( x_f \) and \( x_w \). For gaseous diffusion this is equivalent to electrical energy"

\# of SWU's produced by an enrichment plant
during a time period $t$,

$$SWU = \left[ P \sum V(x_p) + W \sum V(x_w) - F \sum V(x_f) \right] t$$

The quantity $V(x_i)$ is called the separation potential and is given by;

$$V(x_i) = (2x_i - 1) \ln \left( \frac{x_i}{1 - x_i} \right)$$

where $i = f, p, w$

We normally quote SWU's per unit of product (P·t) where P is feed rate.

$$S = \frac{SWU}{P \sum t} = V(x_p) + \left( \frac{W}{P} \sum V(x_w) \right) - \left( \frac{F}{P} \sum V(x_f) \right)$$

$S$ = "SWU" factor, $\frac{SWU}{kg}$

Figure 3.6 plus Schematic

Problem -1
a.) What is the number of kgs of natural U that has to be provided as feed in an enrichment plant if one requests 30,000 kg of U enriched to 3% in $^{35}\text{U}$? Assume the tails assay is 0.2%.

b.) What is the number of SWU’s needed for separation?

\[
\frac{F}{P} = \frac{(3 - 0.2)}{(0.711 - 0.2)} = 5.479 \ \frac{\text{kg feed}}{\text{kg product}}
\]

Total feed is then;

\[
F = 30,000 \cdot 5.479 = 164,370 \ \text{kg U feed}
\]

\[
b.) \ V(x_f) = (2 \sum 0.00711 - 1) \ln \left[ \frac{0.00711}{1 - 0.00711} \right] = 4.869
\]

\[
V(x_w) = (2 \sum 0.002 - 1) \ln \left[ \frac{0.002}{1 - 0.002} \right] = 6.188
\]

\[
V(x_p) = (2 \sum 0.03 - 1) \ln \left[ \frac{0.03}{1 - 0.03} \right] = 3.268
\]

\[
S = 3.268 + (5.479 - 1)(6.188) - (5.479)(4.869) = 4.307
\]

Hence the total number of SWU’s is then;

\[
30,000 \ \text{kg} \cdot 4.307 \ \text{SWU/kg} = 129,210 \ \text{SWUs}
\]
Table 1.1
Summary of Fuel Characteristics in Fission Power Plants
(After Benedict-1981)

<table>
<thead>
<tr>
<th></th>
<th>BWR</th>
<th>PWR</th>
<th>HTGR</th>
<th>CANDU</th>
<th>LMFBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MW(e)</td>
<td>1100</td>
<td>1100</td>
<td>330</td>
<td>508</td>
<td>1200</td>
</tr>
<tr>
<td>Thermal Eff.-%</td>
<td>33</td>
<td>33</td>
<td>39</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>Assembly Geometry</td>
<td>8x8</td>
<td>9x9</td>
<td>17x17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assembly Length-m</td>
<td>3.8</td>
<td>3.7</td>
<td>0.78</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td># of Assemblies</td>
<td>590</td>
<td>180</td>
<td>1482</td>
<td>4680</td>
<td>360</td>
</tr>
<tr>
<td>Core Ht-m</td>
<td>3.8</td>
<td>3.7</td>
<td>4.75</td>
<td>5.95</td>
<td>1</td>
</tr>
<tr>
<td>kg Fuel/assembly</td>
<td>270</td>
<td>600</td>
<td>22</td>
<td>37</td>
<td>80</td>
</tr>
<tr>
<td>Tot.tonne fuel in core</td>
<td>138</td>
<td>90-100</td>
<td>0.77-(^{235})U(^{16-232})Th(^a)</td>
<td>105</td>
<td>29</td>
</tr>
<tr>
<td>BU-MWd per MTU</td>
<td>30,000</td>
<td>30,000</td>
<td>100,000</td>
<td>8,000</td>
<td>100,000</td>
</tr>
<tr>
<td>% Fuel Replaced/y</td>
<td>25</td>
<td>33</td>
<td>18</td>
<td>continuous</td>
<td>Varied</td>
</tr>
<tr>
<td>Enrichment-%</td>
<td>1.8</td>
<td>2.8</td>
<td>93</td>
<td>0.711</td>
<td>15-20 (^{239})Pu</td>
</tr>
<tr>
<td>Power Density-(kW/liter)</td>
<td>54</td>
<td>100</td>
<td>8</td>
<td>12</td>
<td>280</td>
</tr>
<tr>
<td>Linear Ht Rate-kW/m)</td>
<td>19</td>
<td>17</td>
<td>8</td>
<td>26</td>
<td>29</td>
</tr>
<tr>
<td>a-Initial Loading</td>
<td></td>
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</table>

1.) An enrichment plant has a throughput of 32,000 kgU/day and produces 26,000 kgU as tails. What is the enrichment of the product if the feed is natural U and the tails are 0.25%?

2.) A gaseous diffusion method has been proposed to produce BF$_3$ enriched to 90% in B$_{10}$. How many kgs of BF$_3$ feed (natural B) are needed to produce 1 kg of B$_{10}$ with 8% tails?

3.) Calculate the natural U feed and SWU factors 1 billion years into the future. Assume tails of 0.15% and 3% enriched product;

\[
t_{1/2} (^{35}\text{U}) = 7.1 \times 10^8 \text{ y},
\]
\[
t_{1/2} (^{38}\text{U}) = 4.51 \times 10^9 \text{ y}.
\]

4.) Assuming that the price per SWU is $80 and the cost of conversion is $4/kgU, what is the price of the U$_3$O$_8$ ($\text{lb U}_3\text{O}_8$) beyond which it will cost less to enrich the already mined, purified, and converted (to UF$_6$) tails that contain 0.2% $^{35}\text{U}$ rather than mine new U?

[Assume the product will be 3% enriched in either case and the new tails will be 0.1%(when the old tails are enriched). Tails stored as UF$_6$ cost nothing.]