Enrichment and Conversion of Fission Reactor Fuel Elements

- Two fissile isotopes commonly considered:
  - $^{235}\text{U}$ (Use enrichment)
  - $^{239}\text{Pu}$ (Use reprocessing)

- U.S. (weapons---->submarines---->civilian )
  1944 present

- Canada..Commercial..Heavy Water + Nat. U

Gaseous Diffusion Process - $\text{UF}_6$

- Relies on the fact that $\text{UF}_6$ is solid at RT and a vapor at moderate temperatures.

(Figure)

- Gaseous diffusion relies on the difference in the rate which $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$ diffuse through a barrier containing many holes.

- The relative speed of the two molecules can be derived from their kinetic energies:

$$kT = \frac{MV^2}{2} \quad M = \text{molecular mass}$$

or,

$$\frac{V_L}{V_H} = \sqrt{\frac{M_H}{M_L}} = \alpha$$
UF₆ Phase Diagram

Gaseous Diffusion Process Operating Region

- Solidify
- Liquify
- Condense
- Vaporize
- Triple Point
- Gaseous Diffusion Process Operating Region

Pressure (psia)

Temperature (°F)
- Relative frequency at which molecules of different species pass through a small hole is proportional to the speed of the molecule.

Hence the ratio of $\frac{^{35}U}{^{38}U}$ on the low pressure side is greater than the $\frac{^{35}U}{^{38}U}$ ratio on the high pressure side.

*For $^{235}UF_6$ and $^{238}UF_6$, maximum $\alpha$ is;*

$$\alpha = \sqrt{\frac{238 + 6 \cdot 19}{235 + 6 \cdot 19}}$$

$$\alpha = 1.004289$$

( more realistic value is 1.003 due to down stream back pressure and leaks)

- Low value of $\alpha$ requires a very large number of steps

  Figures (2)

- **What is the Barrier?** *(mostly classified)*
  - Very thin and delicate
  - 100’s of millions of holes/cm$^3$
  - $\approx$ 20 Å diameter hole
  - Must exclude organic materials and air to avoid plugging
  - Materials reported to be sintered Ni and anodized Al
Fig. 3.4 A typical gaseous diffusion cascade for enriching and stripping. Note that the pressure is less than atmospheric in all stages in order to increase the mfp of the mole.
GASEOUS DIFFUSION OPERATIONS

Separation At The Normal Feedpoint

To Product Withdrawal

A Stream

Stage N+1

Stage N

Stage N–1

Stage N–2

B Stream

Compressor

Motor

Motor

Normal Feed

A Stream

B Stream

To Depleted Withdrawal (0.2–0.4%)
More complete analysis of U enrichment, see;


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Important Variables and Equations

\[
\text{kg U Feed (F)} = \text{kg Enriched U Product (P)} + \text{kg U Waste (W)}
\]

\[
x_f F = x_p P + x_w W
\]

where

- \(x_f\) = wt. fraction of \(^{35}U\) in feed
- \(x_p\) = wt. fraction of \(^{35}U\) in product
- \(x_w\) = wt. fraction of \(^{35}U\) in waste

(Note: F, P, & W could be in kg or kg/unit time)

• 2 eqs. and 6 variables, F, P, W, \(x_f\), \(x_p\), \(x_w\)
• Trick is to solve for 2 in terms of the other 4!

1.) \(x_f = 0.711\%\) now (1996)
2.) \(x_p\) = as requested by the customer
   Table 1.1 and 2 figures

3.) \(x_w\) = could be between 0.2 and 0.3 %, currently in the U.S. is 0.3%
<table>
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<th>BWR</th>
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<th>CANDU</th>
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<td>Thermal Eff.-%</td>
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<td>Assembly Length-m</td>
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<td># of Assemblies</td>
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<td>kg Fuel /assembly</td>
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<td>600</td>
<td>22</td>
<td>37</td>
<td>80</td>
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<td>Tot.tonne fuel in core</td>
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<td>0.77-235U</td>
<td>16-232Tha</td>
<td>105</td>
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<td>BU-MWd per MTU</td>
<td>30,000</td>
<td>30,000</td>
<td>100,000</td>
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<td>% Fuel Replaced/y</td>
<td>25</td>
<td>33</td>
<td>18</td>
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<td>Enrichment-%</td>
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<td>Linear Ht Rate-kW/m)</td>
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<td>a-Initial Loading</td>
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Figure 7. Average Annual Equilibrium-Cycle Discharge Burnup for U.S. Boiling-Water Reactors, 1972-2010


Figure 8. Average Annual Equilibrium-Cycle Discharge Burnup for U.S. Pressurized-Water Reactors, 1972-2010

4.) \( P = \) mass of desired product

One can solve the equations above;

\[
F = P \left( \frac{x_p - x_w}{x_f - x_w} \right)
\]

\[
W = P \left( \frac{x_p - x_f}{x_f - x_w} \right)
\]

Feed factor is defined as;

\[
\frac{F}{P} = \frac{x_p - x_w}{x_f - x_w}
\]

Waste factor;

\[
\frac{W}{P} = \frac{F}{P} - 1
\]

How Much Energy is Required to Reach a Given Enrichment?

Define Separative Work Unit (SWU) as;

"resource required to perform the enrichment to the desired level of \( x_p \) given \( x_f \) and \( x_w \). For gaseous diffusion this is equivalent to electrical energy"

# of SWU’s produced by an enrichment plant
during a time period $t$,

$$SWU = \left[ P \cdot V(x_p) + W \cdot V(x_w) - F \cdot V(x_f) \right] t$$

The quantity $V(x_i)$ is called the separation potential and is given by:

$$V(x_i) = (2x_i - 1) \ln \left[ \frac{x_i}{1 - x_i} \right]$$

where $i = f, p, w$

We normally quote SWU's per unit of product ($P \cdot t$) where $P$ is feed rate.

$$S = \frac{SWU}{P \cdot t} = V(x_p) + \left( \frac{W}{P} \right) \cdot V(x_w) - \left( \frac{F}{P} \right) V \cdot (x_f)$$

$S = \text{"SWU" factor, } \frac{SWU}{kg}$

Figure 3.6 plus Schematic
Fig. 3.1 Phase diagram of uranium hexafluoride ($\text{UF}_6$).

Number of SWU's needed, as a function of enrichment, for enrichment less than 4%

tails = 0.2%, natural uranium feed

Fig. 3.6 SWUs versus product enrichment; number of SWUs needed, as a function of enrichment, for enrichments of <4%.
SEPARATIVE WORK & TAILS ASSAY ANALOGY

MAKING ONE GALLON OF APPLE CIDER

USUAL OPERATION

FORCE:
Force is like Separative Work

FEED:
1 Bushel of Apples

WASTE:
Peels, Cores, Seeds & Apples

PRODUCT:
1 Gallon of Cider

MORE FORCE, BUT LESS FEED

NEED LESS FEED:
Less Than 1 Bushel of Apples

LESS WASTE:
Fewer Peels, Cores, Seeds & Apples

PRODUCT:
1 Gallon of Cider

MORE FEED, BUT LESS FORCE

USE MORE FEED
More Than 1 Bushel of Apples

MORE WASTE:
More Peels, Cores, Seeds & Apples

PRODUCT:
1 Gallon of Cider
Problem -1

a.) What is the number of kgs of natural U that has to be provided as feed in an enrichment plant if one requests 30,000 kg of U enriched to 3% in $^{35}U$? Assume the tails assay is 0.2%.

b.) What is the number of SWU's needed for separation?

$$F = \frac{(3 - 0.2)}{(0.711 - 0.2)} = 5.479 \frac{kg\text{ feed}}{kg\text{ product}}$$

_Total feed is then;

$$F = 30,000 \cdot 5.479 = 164,370 \text{ kg U feed}$$

b.)

$$V(x_f) = (2 \cdot 0.00711 - 1) \ln \left( \frac{0.00711}{1 - 0.00711} \right)$$

= 4.869

$$V(x_w) = (2 \cdot 0.002 - 1) \ln \left( \frac{0.002}{1 - 0.002} \right)$$

= 6.188

$$V(x_p) = (2 \cdot 0.03 - 1) \ln \left( \frac{0.03}{1 - 0.03} \right)$$

= 3.268

$$S = 3.268 + (5.479\cdot1)(6.188) - (5.479)(4.869) = 4.307$$

_Hence the total number of SWU's is then;

$$30,000 \text{ kg} \cdot 4.307 \text{ SWU/kg} = 129,210 \text{ SWUs}$$
Problems Due Friday, Sept. 24, 1999

1.) An enrichment plant has a throughput of 32,000 kgU/day and produces 26,000 kgU as tails. What is the enrichment of the product if the feed is natural U and the tails are 0.25%?

2.) A gaseous diffusion method has been proposed to produce BF\(_3\) enriched to 90% in B\(^{10}\). How many kgs of BF\(_3\) feed (natural B) are needed to produce 1 kg of B\(^{10}\) with 8% tails?

3.) Calculate the natural U feed and SWU factors 1 billion years into the future. Assume tails of 0.15% and 3% enriched product;

\[
t_{1/2} (^{35}\text{U}) = 7.1 \times 10^8 \text{ y},
\]
\[
t_{1/2} (^{38}\text{U}) = 4.51 \times 10^9 \text{ y}.
\]