MOTIVATIONS

The origin and evolution of planetary systems (such as our solar system), galaxies, and the universe itself is fundamentally intertwined with the relative abundances of the elements and our ability to understand how the present relative abundances came about.

Today, matter is continuously recycled from the interstellar medium (ISM) through stars (*s) and back to the ISM. However, each return to the ISM is accompanied by an enrichment of the matter in elements heavier than H and He due to nuclear processing in the cores of stars.

- Star formation --------------------- Ambient interstellar gas and dust is incorporated into stars.
- Stellar nuclear processing --------- Stars fuse light nuclei into heavier nuclei to produce the stellar luminosities. This also enriches the material in heavier elements at the expense of H and other lighter elements.
- Stellar death ---------------------- Massive stars die a violent death in which they literally explode (go supernova), thereby returning processed material back to the ISM.

Thus, each generation of stars enriches the ISM with heavier elements and over time, the gas in the ISM and in stars becomes more and more abundant in heavier elements that are synthesized in the last generation of stars.

This, at least can account for some of the elements, but not all of them. To understand the whole picture, we have go to back to the Big Bang and trace the formation of the elements from the beginning.

First, let us see what the cosmic relative abundance of the elements looks like.
Figure 1.1. Elemental Abundances of the SAD, (Anders and Grevesse 1989.)

7.4. The SAD and Nuclear Processes: Overview

Figure 7.6. The SAD: Abundances vs. Mass Number (Cameron 1982, with permission).
Note: 1) The most abundant element in the universe by far is H, followed by He (about 10% that of H).
2) All other elements are at least $10^{-3}$ below that of hydrogen.
3) The abundances of Li, Be, and B are very low relative to H and He and even to C and O.
4) From C and O, there is a steady decline in the abundances of the heavier elements with mass (or atomic number).
5) There is a local peak for Fe, Cr, Mn, and Ni. This is called the Fe peak.

This is approximately the relative abundances one would find in the Sun and other stars and the ISM. This is quite different from the abundances of the elements in the Earth’s crust.

We obviously live in an environment much different from that which is representative of most of the universe.

ORIGIN OF THE LIGHT ELEMENTS
Let us now consider the origin of the light elements hydrogen, deuterium, helium, lithium, beryllium, and boron.

In the late 1940’s, George Gamow and his students were considering the implications of the Big Bang with regard to the possibility that the elements might have been formed at the birth of the universe itself. Their reasoning was that if the universe was hot enough and dense enough to support nuclear reactions, then maybe all the elements in essentially their present relative abundances were formed at the beginning of the universe. They began a program to calculate the relative abundances that could be produced by the Big Bang. One of the great insights that came out of this effort was the realization that the Big Bang should have produced a radiation field which filled the universe and which today should be isotropic (the same in all directions) and only a few degrees above 0K. That is, evidence for the Big Bang should be observable today by the redshifted remnant of the early radiation field of the universe. They, in fact, estimated the temperature of what we now call the cosmic background radiation (CBR) to be about 5 K; recall, that the measured value is about 3K. This was published in a classic paper by Alpher, Bethe, and Gamow 1948, Phys. Rev., 73, 803. Bethe actually had nothing to do with the paper, but it suited Gamow’s sense of humor to have the authors in the order of the first 3 letters of the Greek alphabet, so he invited Hans Bethe to join the paper as a coauthor. This was an extremely important and prescient insight, but oddly enough was ignored for more than 15 years. The other major result that came out of this work was the problem of mass 5 and mass 8 nuclei. It turns out that these nuclei are unstable and will decay into lighter nuclei on short time scales. This represented a serious problem for synthesizing heavier nuclei because atoms with these masses represented a canyon over which no one could see how to build a bridge at that time. About the same time, it became clear that stars can also synthesize heavier elements from lighter atoms. Because of this, attention was turned to understanding the synthesis of elements in stars and the Big Bang was largely ignored for several years as a possible important contributor in this arena.

Finally, in the 1970’s it became clear that stars could not account for the hydrogen, deuterium, helium, and lithium in the universe. Stars can only destroy H and D. While they do produce a lot of helium, they cannot account for the huge abundance of this element observed in the universe. In the 1970’s Robert Wagner, David Schramm and a few other cosmologists with expertise in cosmology and high energy physics
calculated nucleosynthesis models in an expanding universe (with accompanying decreasing temperature and density). Their calculations predicted relative abundances of the light elements $H$, $^2H$, $^4He$, $^3He$, Be, and Li.

![Diagram showing nucleosynthesis over time and temperature](image)

**Figure 26-1** Nucleosynthesis in the Big Bang. The top axis gives the age of the Universe; the bottom axis gives temperature; the vertical axis is abundance in terms of the fraction of total mass. *(Adapted from a diagram by R. V. Wagoner)*

The abundances of $^3He$ and $^4He$ relative to $H$ are determined by the ratio of neutrons to protons which is a decreasing function of time because of the decreasing temperature of the universe. As we discussed earlier, the final value of this ratio determines the abundance ratio of helium to hydrogen that can be produced in the Big Bang. In fact, the relative abundances of H, D, He, and Li are quite consistent with the present abundances of these elements in the Sun, other stars, and the interstellar medium.

Returning to overhead #1 in the light of overhead #3, it now becomes obvious why the Li, Be, and B are so low relative to the heavier elements C, N, O, etc. It is now clear that the light elements H, D, He, Li, Be, and B were either in part or entirely formed in the initial few minutes of the birth of the universe itself. Hydrogen and deuterium were produced in the Big Bang and have since this time slowly decreased by being converted into heavier elements by nucleosynthesis in stars. H is the basic fuel that feeds stars. It is converted to He via two possible reaction networks, as I will show below. Although He is formed in stars, the stellar contribution is a minor fraction of the huge amount produced in the Big Bang. Although there are a few possible ways to form Li, Be, and B, stellar nuclear processes burn these elements. These 3 elements have low
cosmic abundances and are present only because they were produced in very small abundances in the Big Bang.

**THE ELEMENTS HEAVIER THAN BORON**

The Big Bang could not produce significant amounts of carbon and heavier elements basically because the universe became too cool to produce these elements by the time it had synthesized their progenitors. The process was also slowed down by the instability of elements of mass 5 and 8 amu.

Where then did the heavier elements come from? To address this question, we must first consider the possible energy sources available to a star.

**A. Chemical and Gravitational Energy**

Exactly how stars can produce the prodigious amounts of energy required to account for their luminosities for billions of years was a deep and serious problem for astrophysicists through the first half of this century. One can easily show that normal chemical burning cannot supply the sun’s luminosity for more than a few thousand years at best and therefore cannot be a significant contributor to the Sun’s luminosity (and, by implication, other stars as well). See overhead #5 for an example of this calculation.
It had earlier been suggested by von Helmholtz (1854) and Lord Kelvin (1861) that the Sun might derive its luminosity from gravitational energy. From basic physics, it has been shown that when a gas contracts due to gravity half of the gravitational potential energy is converted to heat or kinetic energy. Using this result one can calculate the energy/gram of mass available for conversion to heat. This is:

\[ \varepsilon_g = \frac{E_g}{m} = \frac{GM}{2R} \approx 9.54 \times 10^{14} \text{ erg/gm} \]

The rate at which the Sun uses energy/gram of matter is:

\[ \varepsilon_o = \frac{L_o}{M_o} = 2 \text{ erg/gm sec} \]

From these we can show that the Sun could shine at its present rate for a maximum time of:

\[ \tau = \frac{M_H}{M_{H_2}} = \frac{1.5 \times 10^{33} \text{ gm}}{7.58 \times 10^{20} \text{ gm/s}} = 1.98 \times 10^{12} \text{ s} \]

\[ \approx 63,000 \text{ yr.} \]

This is the absolute maximum time for chemical burning.

Age of the Sun \( \geq 4.6 \times 10^9 \text{ yr} \)

Also, the Sun is too hot for \( H \rightarrow H_2 \) to form!

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**Chemical Burning**

If could convert all \( H \rightarrow H_2 \)

Energy of formation \( \approx 11 \text{ eV} \approx 1.76 \times 10^{-11} \text{ erg} \)

To provide solar luminosity via \( H_2 \) formation

No. \( H_2 \) formations/s required is:

\[ N_{H_2} = \frac{L_o}{E_{H_2}} = 2.27 \times 10^{44} \text{ H}_2 / \text{s} \]

Mass of \( H \) burned/s = \( m_{H_2} \)

\[ N_{H_2} = 7.58 \times 10^{20} \text{ gm/s} = M_{H_2} \]

Mass available:

\[ M_H = \frac{M_{H_2}}{M_o} \]

\[ = 0.75 M_o \approx 1.5 \times 10^{33} \text{ gm} \]

Length of time the Sun could shine at its present luminosity by \( H \rightarrow H_2 \):

\[ \tau_{H \rightarrow H_2} = \frac{M_H}{M_{H_2}} = \frac{1.5 \times 10^{33} \text{ gm}}{7.58 \times 10^{20} \text{ gm/s}} = 1.98 \times 10^{12} \text{ s} \]

\[ \approx 63,000 \text{ yr.} \]

This is the absolute maximum time for chemical burning.

Age of the Sun \( \geq 4.6 \times 10^9 \text{ yr} \)

Also, the Sun is too hot for \( H_2 \) to form!
\[ \tau_{KH} = \frac{\varepsilon_g}{\varepsilon_c} \approx 4.77 \times 10^{14} \text{s} \approx 1.5 \times 10^7 \text{yr} \]

This is known as the Kelvin-Helmholtz time. Although long on a human scale, this is a very short time on the time scale of stars and we conclude that gravity cannot be a major contributor to the energy budget of stars. It is, however, the prime source of energy for stars during their formation before a protostar’s core becomes hot enough and dense enough to sustain nuclear reactions. So during the formation phase of a star, this is the main source of energy, over the lifetime of a star it is an insignificant energy source.

B. *Nuclear Energy*

We are, therefore, left with nuclear reactions as the only viable source of energy for stars. Let us now examine this energy source for stars. As has already been mentioned, nuclear reactions provide at least a million times more energy per reaction than chemical reactions. Such a large energy per reaction makes nuclear energy a likely source for stars.

a) Fission

There are two possible mechanisms to obtain energy from nuclear reactions. One is fission which is energy obtained from the breakdown of heavy unstable atoms such as Uranium, Plutonium, Thorium, etc. to lighter atoms. The problem with fission is that unstable nuclei such as $\text{U}^{238}$ have long half lives (~a billion years). To supply 2 erg/gm sec, the Sun would have to be composed essentially entirely of Uranium and Thorium and their decay products. However, the mean density of the Sun is only about 1.4 gm per cubic cm. (about the density of water). The Sun cannot have even a significant fraction of its mass composed of such heavy elements and have this mean density. Thus fission cannot be a viable energy source for stars!

b) Fusion

Nuclear fusion is the process of building up heavier atomic nuclei from lighter ones. We know from experiments with nuclear reactors and from theoretical calculations that it requires a few Mev of energy to overcome the coulomb barrier (i.e. the repulsive force of two like charged protons) so that they can come into contact and fuse with each other. In the core of the Sun, we also know from several lines of reasoning that the temperature is only about 15 million degrees which corresponds to thermal energies of only a few kev. As a consequence, there does not appear to be
enough kinetic energy for protons to get close enough to each other for fusion to occur. Certainly not enough in a classical sense. However, quantum mechanics helps out with a bit of magic called “tunneling” in which a tiny fraction of all near collisions results in the two particles “tunneling” through the coulomb barrier and fusing. The higher the temperature the greater the probability that tunneling will occur and the higher the density the larger the number of collisions per second. See the illustration of this process:
One can show that his process can account for the luminosity of the Sun, even though the fraction of collisions that result in fusion is a tiny fraction of the total number that would occur in the absence of coulomb barriers.
Exactly how do stars produce the element abundances that we have observed? To answer this question, we will examine the nuclear reaction networks active in stars as a function of stellar mass.

**i) low mass stars (mass < 2 solar masses)**

Stars with mass less than about 2 solar masses have central temperatures less than about 20 million degrees K. For temperatures this low the only type of hydrogen burning possible is the proton-proton (P-P) chain illustrated in

![Hydrogen Burning](image)

### Hydrogen Burning

#### I. Proton - Proton Chain

<table>
<thead>
<tr>
<th>Source</th>
<th>Temperature</th>
<th>Life Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>von Weizsäker (1937)</td>
<td>$T \sim 10^7 \text{ - } 2 \times 10^7 \text{ K}$</td>
<td></td>
</tr>
<tr>
<td>Bethe &amp; Critchfield (1938)</td>
<td>$\tau \sim 10^{10} \text{ yr}$</td>
<td></td>
</tr>
</tbody>
</table>

#### A. $\text{He}^3 + \text{He}^4 \rightarrow \text{Be}^7 + \gamma$

- $\text{Be}^7 + \text{e}^- \rightarrow \text{Li}^7 + \nu$
- $\text{Li}^7 + \text{H}^1 \rightarrow \text{He}^4 + \text{He}^4$

$17.347 \text{ MeV}$

#### B. $\text{He}^3 + \text{He}^4 \rightarrow \text{Be}^7 + \gamma$

- $\text{Be}^7 + \text{e}^- \rightarrow \text{Li}^7 + \nu$
- $\text{Li}^7 + \text{H}^1 \rightarrow \text{He}^4 + \text{He}^4$

$12.859 \text{ MeV}$

#### C. $\text{Be}^7 + \text{H}^1 \rightarrow \text{B}^8 + \gamma$

- $\text{B}^8 \rightarrow \text{Be}^8 + \text{e}^- + \nu$
- $\text{Be}^8 \rightarrow \text{He}^4 + \text{He}^4$

$0.095 \text{ MeV}$

Branches B + C require a significant amount of $\text{He}^4$ to be present $\Rightarrow$ only important in later stages. The P-P chain dominates the energy production in the Sun.

Note that the rate of these reactions are very sensitive to the temperature ($\propto T^4$) and only operate at temperatures $>20$ million degrees K. Also, branches II and III are only important in the later stages when there is a significant amount of Be, Li, and B available. The P-P chain is the primary energy source in the Sun. Low mass stars have two fundamental problems.
as far as production of the observed heavy elements is concerned. One, they never become hot enough in their cores to burn elements heavier than He which means that they cannot make elements much heavier than C, O, or Ne, depending on their precise mass. Two, and more to the point, the heavy elements produced in low mass stars are trapped in the core of the star and cannot be released back to the ISM. This is because the atoms produced by nuclear burning in these stars do not go through an explosive demise which could return the processed matter to the ISM. Instead, these elements die with the star and will be tied up in a super dense white dwarf (the final evolution state of low mass stars). Thus, although low mass stars are active producers of He and C, and perhaps some O and Ne, they do not enrich the ISM with many heavy elements.

ii) High mass stars (>2 solar masses)

Massive stars can achieve very high central temperatures and densities due to their large massess. These are stars whose central temperatures are >20 million degrees and can therefore ignite an alternative set of hydrogen burning reactions called the CNO cycle which is illustrated in
II. The CNO Cycle

von Weizsäker (1938) \( T \geq 2 \times 10^7 \) K
Bethe (1939) \( \tau \sim 10^7 \) yr

\[
\text{rate} \propto T^{15}
\]

A. \( ^{12}\text{C} + ^1\text{H} \rightarrow ^{13}\text{N} + \gamma \quad 1.944 \text{ MeV} \\
^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu \quad 2.221 - 0.710 \\
^{13}\text{C} + ^1\text{H} \rightarrow ^{14}\text{N} + \gamma \quad 7.550 \\
^{14}\text{N} + ^1\text{H} \rightarrow ^{15}\text{O} + \gamma \quad 7.293 \\
^{15}\text{O} \rightarrow ^{15}\text{N} + e^+ + \nu \quad 2.761 - 1.000 \\
^{15}\text{N} + ^1\text{H} \rightarrow ^{12}\text{C} + ^4\text{He}^4 \quad 4.965
\]

B. \( ^{15}\text{N} + ^1\text{H} \rightarrow ^{16}\text{O} + \gamma \quad 12.126 \text{ MeV} \\
^{16}\text{O} + ^1\text{H} \rightarrow ^{17}\text{F} + \gamma \quad 0.601 \\
^{17}\text{F} \rightarrow ^{17}\text{O} + e^+ + \nu \quad 2.762 - 0.94 \\
^{17}\text{O} + ^1\text{H} \rightarrow ^{14}\text{N} + ^4\text{He}^4 \quad 1.193
\]

CNO operates at a higher T than P-P because the coulomb barriers of C,N, + O are higher than H,D, + He³.

C,N, + O simply act as catalysts but are not used up. Does convert \( ^{15}\text{N} \rightarrow ^{14}\text{N} \).

Note that the rate at which these reactions occur is extremely sensitive to temperature \((\propto T^{15})\) so it is much much faster than the P-P chain. Although this set of reactions occur in more massive stars which have more fuel to burn, these stars have luminous lifetimes a factor of about 1000 times shorter than the Sun because their fuel is burned a factor of about 10,000 times faster than in the Sun. The reason the CNO cycle requires higher temperatures is because the coulomb barriers of C, N, and O are much greater than that of H, D, and He (due to their higher nuclear charges).

The choice of which type of hydrogen burning (P-P Chain or the CNO Cycle) dominates in the core of a star is determined entirely by the central temperature of the star. This is illustrated in
Up to this point we have shown that the cosmic He can be enriched by stars, but what about the elements heavier than He? The elements available to a star for nuclear fuel is determined entirely by the mass of the star which in turn determines the maximum central temperature that it can achieve. For stars that are massive enough to achieve a central temperature $\geq 100$ million degrees, not only can He burning be ignited (alpha captures), but also carbon-burning and oxygen-burning can occur via the reactions shown in

![Graph showing CNO bi-cycle and PP chain](image-url)
## Helium Burning

**Triple Alpha Process**  
\[ T \gtrsim 10^8 \text{K} \]  
\[ \tau \lessgtr 10^7 \text{yr} \]

\[
\begin{align*}
\text{He}^4 + \text{He}^4 & \rightarrow \text{Be}^8 & -0.0921 \text{ MeV} \\
\text{Be}^8 + \text{He}^4 & \rightarrow \text{^12C} & -0.286 \\
\text{^12C} & \rightarrow \text{C}^{12} + \gamma & +7.656
\end{align*}
\]

When enough $^{12}\text{C}$ has been formed, then can have $\alpha$-captures to form heavier atoms.

\[
\begin{align*}
\text{C}^{12} + \text{He}^4 & \rightarrow \text{O}^{16} + \gamma & 7.161 \text{ MeV} \\
\text{O}^{16} + \text{He}^4 & \rightarrow \text{Ne}^{20} + \gamma & 4.730 \\
\text{Ne}^{20} + \text{He}^4 & \rightarrow \text{Mg}^{24} + \gamma & 9.317 \\
\text{Mg}^{24} + \text{He}^4 & \rightarrow \text{Si}^{28} + \gamma & 9.981
\end{align*}
\]
Note that the time these reactions can sustain a star is very short (only about a hundred thousand years or less) and the amount of energy derived from them is relatively small compared to that from hydrogen burning. Even more massive stars can ignite heavier element as shown in

Other higher T reactions:

Silicon burning: \( T > 3-4 \times 10^9 \text{ K}; \ \tau \sim 1 \text{ s} \)
S-process (slow): \( T > 10^8 \text{ K}; \ \tau \sim 10^5 - 10^7 \text{ yr} \)
r-process (rapid): \( T > 10^{10} \text{ K}; \ \tau = 10 - 100 \text{ s} \)
p-process: \( T > 2-3\times10^9 \text{ K}; \ \tau = 10 - 100 \text{ s} \)
What happens when one source of fuel is exhausted in a massive star is that the core stops nuclear reactions and as a consequence starts to contract since there is no energy generation to produce the internal pressure necessary to support the overlying layers. As the core contracts it heats up from the conversion of gravitational potential energy to heat. This will continue until the core or layer concerned reaches a temperature high enough to ignite the next possible source of fuel (carbon, oxygen, silicon, s-process, r-process, e-process, etc.). In the latest stages of a massive star’s life, it will have an onion like structure with an inert pure Fe core, surrounded by successively larger but thin shells of silicon burning, oxygen-burning, neon burning, carbon burning, helium burning, and a hydrogen burning shell. Above the hydrogen burning shell the star will have a huge low density envelope almost as big as Jupiter’s orbit around the Sun. This is illustrated in:
A massive star just prior to going supernova.

Envelope is about the size of Jupiter’s Orbit.

Core is about the size of the Earth.

**Figure 10.3** Shells in the core of a high-mass star as it evolves away from the main sequence. (a) The core is only a small fraction of the total radius. (b) In the core, there is a succession of shells of different composition. Each shell has exhausted the fuels that are still burning in shells further out.
Once the star has formed an Fe core it can not derive any further energy from this material because Fe-56 nuclei are so tightly bound that no further energy can be obtained by fusing other nuclei with it. It requires more energy to fuse other nuclei with Fe-56 than comes out of the reaction. This is illustrated in

Thus Fe-56 is the end produce (ash) of nuclear fusion processes in the most massive stars which achieve high enough central temperatures to burn nuclei up to iron.

Ultimately, this bloated massive star with a very dense (nearly degenerate) Fe core will finally exhaust all the nuclear fuel available to it in the overlying layers and the whole thing will collapse toward the inert core because it cannot generate enough internal pressure to support itself. Eventually the core will become so dense that it becomes neutron degenerate and is so stiff that it can stop the contraction and the outer processed layers bounce outward with explosive force and luminosity. The entire star explodes and within a fraction of a second, the inert Fe core decomposes into neutrons and the outer shells are explosively expelled in
an extremely luminous supernova which can be as bright as an entire galaxy for a short time.

SLIDES

This explosion returns the heavy elements produced over the past several million years back to the ISM, enriching the local ISM in these elements. The detailed model calculations show an extremely good agreement with the observed abundances today. In fact, the peaks in the relative abundance curve apparent in overhead #2 can be identified with specific nuclear burning processes as shown in

Thus, it is now believed, at least to first order, that the relative abundances of the elements is understood. In summary: H, D, He, and most of the Li, Be, and B were produced in the Big Bang. He has also been supplemented by stellar nucleosynthesis, but this is a small fraction of that produced in the Big Bang. C and heavier elements were produced in massive stars and returned to the ISM by supernovae explosions. The various peaks in the abundance versus atomic number are identified with specific nuclear reaction networks and verify the basic ideas believed to be responsible for synthesis of the elements in stars.