1.) Definitions
\( \alpha, \beta, \gamma, \nu, n, p \)

2.) Mass Energy Relationships

Particles;
\[ E_o = m_o c^2 \]

\( E_o \) is the energy equivalent to rest mass \( (m_o) \) of particle,
\( c = \text{speed of light, } \approx 3 \times 10^8 \text{ m/s} \)

Photons

\[ E = h \nu \]

\( h = \text{Planck’s constant } = 6.626 \times 10^{-34} \text{ joule-s} \)
\( \nu = \text{frequency of radiation, } s^{-1} \)

deBroglie equation,
\[ \lambda = \frac{h}{p} = \frac{h}{mv} \]
\[ = \frac{hc}{E} = 12.4 \frac{\text{Å}}{E(keV)} \]
3.) Radioactive Decay of Unstable Isotopes, $^{A}X_{Z}$ Particles

\[ ^{A}X_{Z} \rightarrow ^{A-4}X_{Z-2} + ^{4}\alpha_{2} \]

(parenct) (daughter)

\[ ^{238}Pu_{94} \rightarrow ^{234}U_{92} + ^{4}He_{2} \]

From nuclear reactor physics we find;

\[ E_{\alpha} = \frac{E}{1 + \left( \frac{4.0026}{A_{d}} \right)} \]

\[ E = Q_{a} + \Delta E \]

Total transitional energy

\[ \Delta E \]

between ground state of parent & daughter

\[ E_{p} \]

Excitation energy of $\alpha$ emitting level in parent

\[ E_{d} \]

Excitation of daughter nucleus fed by decay

Electron Events

Too many n’s

\[ ^{1}n_{o} \rightarrow ^{1}p_{1} + ^{0}\beta_{-1} + \bar{v}_{e} \]

\[ ^{A}X_{Z} \rightarrow ^{A}Y_{Z+1} + ^{0}\beta_{-1} + \bar{v}_{e} \]
Too few n’s

\[ {^1p} \rightarrow {^1n} + {^0\beta} + e \quad \text{AXZ} \rightarrow {^A}Y_{Z-1} + {^0\beta} + e \]

Too few n’s

\[ {^1p} + {^0\beta} \rightarrow {^1n} + e \quad \text{AXZ} + {^0\beta} \rightarrow {^A}Y_{Z-1} + e \]

-----------------------------------------------

Example \(^{238}\text{Pu}_{94}\) decay chain

4.) Law of Radioactive Decay

\[ \frac{dN}{dt} = -\lambda N \]

N = # of radioactive atoms

\[ \lambda = \text{decay constant} = \frac{0.693}{t_{1/2}} \]

\[ t_{1/2} = \text{half life} \]

1 Curie = \(3.7 \times 10^{10}\) disintegrations per second

Can rewrite decay equation:

\[ \int \frac{dN}{N} = \int \lambda \ dt \]
Fig. 1.7 Expanded plutonium-238 decay chain. Courtesy of Los Alamos National Laboratory.
which gives

\[ N = N_0 \exp(-\lambda t) \]

Can also express the instantaneous power released:

\[ Q(t) = Q_0 \exp(-\lambda t) \]

**Problem**

Assume a radioisotope system which contains 1 kg of PuO₂ (90% ²³⁸Pu) which initially operates at 0.4 W/g of ²³⁸Pu. What is the thermal power after 10 years?

A.) \[ t_{1/2} = 87.75 \text{ y} \]

\[ \lambda = \frac{0.693}{87.75} = 0.0079 \text{ y}^{-1} \]

B.)

\[ Q_0 = \{1000 \text{ g PuO}_2 \cdot \left(\frac{238 \text{ g Pu}}{270 \text{ g PuO}_2}\right) \cdot \left(\frac{0.9 \text{ ²³⁸Pu}}{\text{ Pu}}\right)\} \]

\[ \cdot \left[\frac{0.4 \text{ W}}{\text{ g ²³⁸Pu}}\right] \]

\[ = 317 \text{ Watts} \]

C.) After 10 years;

\[ Q = 317 \cdot \exp(-0.0079 \times 10) \]

\[ = 293 \text{ Watts} \]
5.) Cross Sections

Interaction probability for a given reaction by a particle traveling through a given medium;

microscopic, \( \sigma \) \( \text{cm}^2 \) (1 barn = \( 10^{-24} \text{cm}^2 \))

macroscopic, \( \Sigma = N \sigma \text{cm}^{-1} \)

so,

\[ \Sigma \ dx = \text{probability of interaction per unit track length} \]

Removal of particles from a given state;

\[-d\Phi(x) = \Phi(x) \Sigma \ dx\]

Or,

\[ \Phi(x) = \Phi_0 \exp (-\Sigma x) \]

Mean distance before undergoing a reaction:

\[ \lambda = 1/\Sigma \]
6.) Neutron Spectra

When neutrons, which are emitted during the fission process, slow down and are thermalized, they come close to a Maxwell-Boltzmann distribution.

\[ n(E) = \frac{dn}{dE} \left\{ \frac{2\pi n}{(\pi kT)^{\frac{3}{2}}} \right\} \cdot \exp\left( -\frac{E}{kT} \right) \cdot \sqrt{E} \]

Figures 2.12 & 2.13

@ T= 20°C most probable thermal velocity is 2200 m/s which give \( E = 0.0252 \text{ eV} \)

For;

\( E_n < 0.0252 \text{ eV} \) cold neutrons
\( 1 < E_n < 100 \text{ eV} \) resonance n’s
\( 0.1 < E_n < 1 \text{ keV} \) intermediate n’s
\( 0.5 < E_n < 15 \text{ MeV} \) fast neutrons

Lots of Cross Sections!

Figure 2.14

======================================================================
Fig. 2.12 Neutron energy distribution for weakly absorbing medium. Courtesy of Oak Ridge National Laboratory [4].

Fig. 2.13 Neutron energy distribution for highly absorbing medium. Courtesy of Oak Ridge National Laboratory [4].
7.) **Neutron Slowing Down**

Elastic scattering useful for dropping the n energy from the MeV range to the eV range where fissioning is more probable ($\sigma_{\text{fiss}}$ is the highest)

$$E_{\text{min}} = \left[\frac{(A-1)}{(A+1)}\right]^2 E_0 = \alpha E_0$$

$A = \text{At. mass of nuclide}/\text{At. mass of neutron}$

Logarithmic Energy Decrement

$$\xi = \ln\left(\frac{E_0}{E}\right) = 1 + \frac{(\alpha\cdot\ln\alpha)}{(1-\alpha)}$$

Would like high values of $\xi$

See Table 2.3

8.) **Production of Isotopes**

Formation rate $= \Phi \sigma_{\text{act}} N - \lambda N$

where:

$\sigma_{\text{act}}$ is cross section for producing isotope Y from parent X

$N = \text{no. of parent X atoms per cc}.$
Fig. 2.14 Relationship of various neutron cross sections. 
Courtesy of Oak Ridge National Laboratory [4].
<table>
<thead>
<tr>
<th>Material</th>
<th>Atomic Mass (A)</th>
<th>Normal Density (g/cm³)</th>
<th>ξ</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen (H)</td>
<td>1</td>
<td>8.9 × 10⁻⁵ (gas)</td>
<td>1.000</td>
<td>0</td>
</tr>
<tr>
<td>Lithium (Li)</td>
<td>7</td>
<td>0.534</td>
<td>0.268</td>
<td>0.563</td>
</tr>
<tr>
<td>Beryllium (Be)</td>
<td>9</td>
<td>1.85</td>
<td>0.209</td>
<td>0.640</td>
</tr>
<tr>
<td>Carbon (C)</td>
<td>12</td>
<td>1.60</td>
<td>0.158</td>
<td>0.716</td>
</tr>
<tr>
<td>Oxygen (O)</td>
<td>16</td>
<td>0.0014 (gas)</td>
<td>0.120</td>
<td>0.779</td>
</tr>
<tr>
<td>Sodium (Na)</td>
<td>23</td>
<td>0.971</td>
<td>0.083</td>
<td>0.840</td>
</tr>
<tr>
<td>Iron (Fe)</td>
<td>56</td>
<td>7.86</td>
<td>0.0357</td>
<td>0.931</td>
</tr>
<tr>
<td>Tungsten (W)</td>
<td>184</td>
<td>19.2</td>
<td>0.0108</td>
<td>0.979</td>
</tr>
<tr>
<td>Uranium (U)</td>
<td>238</td>
<td>19.1</td>
<td>0.0084</td>
<td>0.983</td>
</tr>
<tr>
<td>Plutonium (Pu)</td>
<td>239</td>
<td>19.6</td>
<td>0.0083</td>
<td>0.983</td>
</tr>
</tbody>
</table>
However, if isotope Y is radioactive, then \( N^* \) is the number of Y atoms per cc at any time.

\[
\text{decay rate} = \lambda \ N^*
\]

Net production of Y:

\[
\frac{dN^*}{dt} = \Phi \sigma_{act} N - \lambda N
\]

Can show that,

\[
N^* = \left[ \left( \frac{\Phi \sigma_{act} N}{\lambda} \right) \right] \cdot \left[ 1 - \exp(-\lambda t_r) \right]
\]

Activity after the neutrons are turned off:

\[
\lambda N^* = \Phi \sigma_{act} N \cdot \left[ 1 - \exp(-\lambda t) \right] \cdot \exp(-\lambda t_r)
\]

Where \( t_r = \) time after removal from the n flux

9.) Some Observations on Reactor Operation

Define \( k = \frac{\# \text{ of fissions (or n’s) in one generation}}{\# \text{ of fissions (or n’s) in the immediately preceding generation}} \)

\( k < 1 \) subcritical
$k = 1$ critical
$k > 1$ supercritical

Figure 3.2

Description of space reactor - Figure 3.3

Fissionable elements - Tables 3.1 & 3.2

Energy released - Table 3.3

Fission Products - Figure 3.4
CHAIN REACTION

(\text{FP} = \text{FISSION PRODUCTS})
MULTIPLICATION FACTOR AS A FUNCTION OF REACTOR CONDITIONS

TIME (NOT TO SCALE)

Fig. 3.2 Multiplication factor as a function of reactor conditions.
GENERIC SPACE NUCLEAR REACTOR SYSTEM

- RADIATION SHIELD
- CORE: CONTAINS FUEL AND MODERATOR*
- CONTROL DRUMS
- REFLECTOR

Coolant IN

Coolant OUT

* THERMAL REACTORS ONLY

Figure 3.3
Table 3.1  Neutron fission thresholds as a function of nuclear mass (calculated).

<table>
<thead>
<tr>
<th>Mass No., A</th>
<th>Fission Threshold (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>18.5</td>
</tr>
<tr>
<td>60</td>
<td>48</td>
</tr>
<tr>
<td>100</td>
<td>47</td>
</tr>
<tr>
<td>140</td>
<td>62</td>
</tr>
<tr>
<td>200</td>
<td>40</td>
</tr>
<tr>
<td>236</td>
<td>(~5)</td>
</tr>
</tbody>
</table>

Table 3.2  Neutron fission thresholds of heavy nuclides (experimental).

<table>
<thead>
<tr>
<th>Target Nucleus</th>
<th>Compound Nucleus</th>
<th>Fission Threshold (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{232}\text{Th}$</td>
<td>$^{233}\text{Th}$</td>
<td>1.3</td>
</tr>
<tr>
<td>$^{233}\text{U}$</td>
<td>$^{234}\text{U}$</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>$^{234}\text{U}$</td>
<td>$^{235}\text{U}$</td>
<td>0.4</td>
</tr>
<tr>
<td>$^{235}\text{U}$</td>
<td>$^{236}\text{U}$</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>$^{236}\text{U}$</td>
<td>$^{237}\text{U}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$^{238}\text{U}$</td>
<td>$^{239}\text{U}$</td>
<td>1.2</td>
</tr>
<tr>
<td>$^{237}\text{Np}$</td>
<td>$^{238}\text{Np}$</td>
<td>0.4</td>
</tr>
<tr>
<td>$^{239}\text{Pu}$</td>
<td>$^{240}\text{Pu}$</td>
<td>&lt; 0</td>
</tr>
</tbody>
</table>
Fig. 3.4  Fission product mass distribution for uranium-235 fission [8].

Table 3.3  Typical energy distribution for Uranium-235 fission.

<table>
<thead>
<tr>
<th>Energy Form</th>
<th>Energy Released (MeV)</th>
<th>Energy Potentially Recoverable (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic Energy of Fission Fragments</td>
<td>168</td>
<td>168</td>
</tr>
<tr>
<td>Decay of Fission Products</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Beta Radiation</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>- Gamma Radiation</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>- Neutrinos</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Prompt (Fission) Gamma Radiation</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Kinetic Energy of Fission Neutrons</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Capture Gamma Radiation</td>
<td></td>
<td>3–12</td>
</tr>
<tr>
<td><strong>TOTALS</strong></td>
<td><strong>207</strong></td>
<td><strong>198–207</strong></td>
</tr>
</tbody>
</table>